THE UNINTENDED CONSEQUENCES OF A LEARNING TRAJECTORIES APPROACH

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This case study, which is a part of a larger design study, examines teachers' uses of learning trajectories (LTs) in diverse classrooms. Specifically data were collected and analyzed for evidence of equitable use of LTs and learning trajectory-based instruction (LTBI). Qualitative analysis revealed that the use of LTBI did not guarantee equitable instructional practices. Moreover, a number of deficit-oriented themes emerged in one case.

Keywords: Learning Trajectories; Equity and Diversity; Teacher Beliefs; Teacher Education-Inservice

Introduction

Research has demonstrated that professional development (PD) that focuses on students' mathematical thinking can lead to changes in teachers' practice (Kazemi & Franke, 2004; Sowder, 2007). Learning trajectories (LTs) are gaining prominence as a representation of students' mathematical thinking, and are thus being used in professional development projects nationwide. While initial results of LT-focused professional development projects are positive, these results have not yet been teased apart to investigate if these benefits are present for traditionally underserved students.

In this paper, I present findings from a case study of teachers that participated in a yearlong professional development project focused on LTs. I begin this report with a brief discussion about learning trajectories. I focus this discussion on how research on LTs has developed over time, specifically in relation to their use in classroom instruction. I follow with a discussion of the theoretical framework that guided this study. I describe the context of the study (location and professional development) and present findings from one case that was particularly interesting and problematic. I conclude this report by discussing implications of the findings.

Background

Over the past two decades, a number of scholars have written about and argued for the use of LTs in mathematics and science education (Battista, 2004; Brown, Clements & Sarama, 2007; Clements & Sarama, 2008; Clements, Wilson & Sarama, 2004; Clements & Sarama, 2008; Confrey et al, 2009; Duncan, Rogat & Yarden, 2009). While the terminology differs (e.g., learning progressions vs. learning trajectories), many of these scholars agree that LTs have the potential to transform the teaching-learning process. LTs have shown promise because they: simultaneously attend to specific skills as well as broader concepts; they attend to how student thinking develops over time (NRC, 2007); they are empirically developed from work with students; and they attend to probable pathways students may take as well as common misconceptions in their mathematical development (Confrey, 2006).

Research on LTs has occurred along three primary fronts: a) developing and validating LTs in different mathematical strands (Battista, 2006; Clements & Sarama, 2007; Maloney & Confrey, 2010); b) using LTs to design curriculum and assessment (NRC, 2007); and c) exploring the ways teachers use LTs in instruction (Bardsley, 2006; Edgington, 2012; Mojica, 2010; Wilson, 2009). Regarding the latter, previous research on LTs has shown promise. Using knowledge of student thinking as represented by LTs, teachers have: a) set goals based on students' developmental level (Clements, 2007); b) described student work with greater detail (Wilson, 2009); c) assessed students

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more effectively (McKool, 2009); and d) anticipated students' strategies as well as misconceptions (Edgington, 2012). What remains unexamined is whether or not *all* students benefit from the promise of LTs as well as how teachers use their knowledge of LTs with traditionally underserved students. Specifically, are LTs used equitably?

Theoretical Framework

To examine whether or not LTs were used equitably in the classroom, I framed this study using Gutierrez (2007) equity framework. In this and other work, she articulates four dimensions of equity: access, achievement, identity, and power. Because Gutiérrez' initial framework was developed from her work with high school math departments and school districts, my first step in this work was to reconceptualize this framework at the classroom level. Additionally, I conjectured ways in which teachers *might* use LTs to promote access, identity, achievement, and power. Table 1 represents a conjecture of the ways in which teachers might use LTs in to promote equitable instruction (Myers, Sztajn, Wilson & Edgington, in review).

Table 1. Conceptual Framework for LTBI and Equity (E-LTBI Framework)

	Teachers use their knowledge of LTs and LTBI to:
Access	Design instruction and instructional tasks such that they are accessible for all students.
	Identify and use up-to-date research based materials and technology
	Be accessible to and attend to all students in the class.
	Foster classroom discussions such that all students can participate and engage.
	Provide all students with opportunities to engage in rigorous mathematics.
Achievement	Set high, yet appropriate, academic standards for all students.
	Unpack and build upon their students' prior mathematical knowledge and use it as a basis for understanding more meaningful and complex mathematics.
	Select and use a variety of forms of assessment (e.g., formative, summative, projects, class discussions) to gauge student achievement.
Identity	Support the development of a robust mathematical identity
	Listen to and consider students out-of-school experiences and design instructional activities that incorporate elements from their homes and communities.
	Validate the use of students' own algorithms and strategies to solve problems.
	Assist students to build connections between the mathematics they learn and the broader world/society.
	Encourage students to engage in mathematical tasks according to their preferences and participate in mathematical discourse in ways that are comfortable for them.
Power	Ensure that students have voice in the classroom.
	Position students as experts in the classroom (this includes things they know in school and things they know from outside of school).
	Allow students to solve problems that are relevant to them (these problems can exist inside or outside of school).
	Encourage all students to present, justify, and defend their mathematical ideas/arguments.
	Help students to see themselves as sources of mathematical knowledge.

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Methods

The Learning Trajectories Based Instruction (LTBI) project is a multi-year design project that examined teacher learning of LTs and how LTs were used in instruction (Sztajn et al., 2012). The participants in this case study represented a subset of teachers in a larger design study. The research question under investigation for the case study was: In what ways do teachers use LTBI to promote access, achievement, identity, and power in their instruction? In this report however, I address a smaller research question: *How do deficit perspectives manifest in the context of an LT-focused professional development project?*

Context

Learning trajectory-based instruction is defined as instruction that situates LTs at the center of the teaching practice (Sztajn et al., 2012). The goals of the larger LTBI project were to (1) explore the impact of LTs on elementary teachers mathematics instruction; (2) build a conceptual model of instruction that is centered on LTs; and (3) confront teachers' stereotypes of students (van Langenhove & Harré, 1999) by focusing on what students can do (as seen in LTs) and building upon it through supportive classroom practices (instead of focusing on what they cannot do).

The professional development began with a 30-hour summer institute in which participants learned about Clements and Sarama's (2009) LTs for early number and counting, addition and subtraction problem types, Smith and Stein's (2008) five practices for mathematical discourse, as well as formative assessment. Because the content of the LT was dense, the LTBI research team organized multiple LTs from Clements and Sarama's work into what we called "Learner Profiles" (Myers, Sztajn, Wilson & Edgington, in review). These profiles—named perceptual child, direct modeling child, counting on child, place value child and multi digit child—provided a way for teachers to "chunk" information about students and created a broader perspective. Teachers participated in an additional 30 hours of PD throughout the school year where they engaged in professional learning tasks related to both the content of the LT and its use in instruction. These tasks included watching clinical interviews, analyzing samples of students' work, curriculum evaluation, and task design. Participants also conducted a number of activities with their own students, which served as the basis for project discussions.

While there were a number of initial conjectures from the project related to teacher learning, discourse, and positioning/stereotypes, three of these conjectures were particularly relevant to this case study. First, the LTBI team conjectured that the learner profiles would be a more manageable grain-size for teachers and offer them a productive way to talk about "where" their students were mathematically. Second, the LTBI team conjectured that LTBI could support teachers' focus on individual students and help teachers design instruction that meets their needs based on their current conceptions. Finally, the LTBI team conjectured that teachers would reduce their use of the words "low" and "high" to describe their students and begin describing their mathematical work using language from the LT.

The partner school for this study was a small elementary school in a suburban district in the southeastern United States. The enrollment was 370 students and the five-acre campus was situated in the heart of a historic district. Demographic data indicated that 36.2% of students are considered economically disadvantaged and 27.1% of students had limited English proficiency.

Participants

Seven teachers participated in the larger design study described above. Because the content of the learning trajectory focused on early number knowledge (including counting), four K-1 case teachers were selected for additional research. Case teachers participated in four interviews and three classroom observation cycles during a four-month period. At the start of this case study, the teachers

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had completed 40 hours of the professional development. The remaining 20 hours of PD occurred concurrently with the case study.

For the purpose of this paper, I focus on one case "Elizabeth." At the time of the study, Elizabeth had 15 years of teaching experience. She was in her fourth year teaching kindergarten and had been teaching at the partner school for nine years. Elizabeth was selected as the focal case for this paper because she demonstrated a deficit orientation throughout the PD project.

Data Sources and Analysis

Data sources for this case study consisted of student mathematical portraits, interviews, classroom observations, and field notes. The mathematical portraits consisted of questions related to how teachers use LTs to: solicit evidence of students' mathematical understandings, set goals for students, and create mathematical opportunities for students. These portraits were created so that the researcher could examine the ways in which teachers talked about individual students as well as groups of students. The mathematical portraits were revisited at each interview during the course of the study. Interview protocols addressed each dimension of the E-LTBI framework and additional questions were added as themes emerged during the study. An observation protocol was also developed to address the E-LTBI framework as well as other aspects of equitable instruction (Author, 2014).

Data analysis occurred in two phases: ongoing and retrospective. The goal of on-going analysis was to understand emergent themes as they related to the E-LTBI framework. All interviews and field notes were reviewed immediately and notes were made regarding preliminary themes. Both predetermined and open coding was used during ongoing analysis. The pre-determined codes consisted of the four dimensions of the E-LTBI framework (access, achievement, identity, and power) as well as the LTBI instructional practices (task/learning goal, anticipate, monitor, select/sequence, and connect). While pre-determined codes resulted from the research questions and the LTBI instructional model, open coding allowed for the emergence of new themes. The following codes were developed during open coding: curriculum, high vs. low students, comparing, literacy, usefulness of LTs, using "high" students as exemplars, deficit orientations, and motivation. During retrospective analysis both within-case and cross-case analysis were conducted (Merriam, 1998).

Results

While a number of themes emerged during the data analysis, one particular manifested throughout Elizabeth's case. At different periods during the study, Elizabeth displayed a deficit perspective towards certain students in her classroom. In some cases, she actually used language from the trajectory to justify her beliefs. Here, I present a number of examples that highlight Elizabeth's deficit orientations as well as how she failed to use LTBI equitably.

Tracking

In my opening interview with Elizabeth, I asked her to talk to me about where each of her students was currently working in the learning trajectory. Elizabeth chose to group her students into four groups. She labeled these groups, "...lower group, next up from low, not quite counting on, and higher group." Later in the study, I again asked Elizabeth to talk to me about her students and where they were working in the trajectory. Although the LTBI team conjectured that teachers would describe groups of students using mathematical evidence from the LT, this was not the case with Elizabeth. Throughout the study, she continued to use language of "low" and "high" and only described her "high" students with LT language.

Not only were these labels of "low" and "high" problematic, but also the demographics of the students that fell into these two groups were alarming. Of Elizabeth's 17 students, eight of them were

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in the two "lower" groups and the remaining nine students were in the "higher" groups. Of the eight students in the lower groups, six were Hispanic, one African-American, and the other was White. Of the nine students in the "higher" groups, seven were White, one African American and one Hispanic. Throughout the course of the study, the only student that moved out of the lower track (according to Elizabeth's assessment) was the one White student. As you will see in the coming examples, Elizabeth used LTBI to provide opportunities for her "higher" students while her "lower" students were denied the benefits of LT-based instruction.

This particular finding is problematic because one conjecture from the project was that teachers could use knowledge of LTBI to focus on individual students and design instruction to move them forward. At the conclusion of the study, Elizabeth indicated that her students in the higher group continued to achieve, while those in the lower group did not.

Access

One aspect of the LTBI PD was a focus on creating high demand tasks with multiple entry points. Teachers had opportunities to examine a variety of tasks and modify them such that students at multiple levels of the LT could access the task. When I asked Elizabeth to explain how she ensures that her tasks are accessible by students at various LT levels, she indicated that the students at the lowest level needed step-by-step instructions. Specifically, she stated:

You take it step-by-step and for each step you plan for them not understanding it possibly. And for the ones it's not accessible because they just don't comprehend it...[make] sure through the activity progressively each step is accomplished and comprehended and then move on to the next step. Check for comprehension at the lowest level and once that's clear, move to the next step.

In this example, Elizabeth indicated that the learning trajectory could be used as a rigid checklist to monitor students' progress. In contrast to our discussions about using LTs develop open tasks that were accessible for students working at different profile levels; Elizabeth indicated that students at the lowest level needed more procedural tasks.

Another important component of access was that teachers would use LTs to design rigorous tasks for students at all LT levels. When I asked Elizabeth about how she used the LT to design these opportunities, she stated:

I remember. ... we do set appropriate academic standards, and I think since our last meeting when I mentioned that we thought that the idea of them counting to 100 and counting to 100 by tens would be pretty much asking too much but we did it every day and now a good majority of them can. Not successful for I'd say 50% of the class but we work it into. . . I work things into my daily routines so they got daily practice, and those that could achieve it at least have the opportunity to try and the opportunity to practice.

In this example, Elizabeth indicated that rigorous goals were appropriate for those that "could achieve." Throughout the study, Elizabeth offered some students opportunities to be challenged mathematically, while those that were "lower on the trajectory" or "floundering" were given rote tasks

Achievement

A focus of the achievement dimension is that teachers could use LTBI to set short- and long-term goals for individual students, assess their progress, and design instruction that builds upon students' individual conceptions as identified in LTs. During a pre-lesson observation, I asked Elizabeth to think about the skills her students had demonstrated in class and how she would build upon students' conceptions. Elizabeth responded:

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They have learned to count to ten, which is great because they didn't do that before. I'm trying to think. They don't have a number sense and...I guess it's hard to say what they do have, honestly, I guess because I worry so much about what they don't have. But they have gained since the very beginning of the year and just being able to count to ten, and even to twenty. Not completely accurately but they know if they put one number after the other, they're seeing things in order. I guess I should be more positive but really they're I don't know how to express it in a positive way. I worry that they don't grasp just the whole idea of numbers and amount, and why it's important.

This example highlights tension Elizabeth felt. While she was able to identify some progress that her students were making, her primary focus was what students did not know. She focused more on skills that the students did not demonstrate and she did not use the LT did not provide her with the agency to develop appropriate instructional plans.

Identity

In the identity dimension, teachers use their knowledge of LTs to solicit and validate various approaches to tasks. Another element of this dimension is that teachers help students build connections between in-school mathematics and out-of-school mathematics. When I asked Elizabeth how she could facilitate this in her classroom, she stated:

I don't think they have the ability to really grasp say, say I think maybe fourth, fifth grade demography and geography. That might be something a little bit more of a mathematical connection but for their level and I'll introduce grander ideas or bigger numbers and they kind of get it but I think my kids who are challenged with second language and also just the beginning of number sense, they can't get it.

In this statement, Elizabeth indicates that English language learners or students developing number sense could not "get it." This statement also highlights that Elizabeth was unwilling to think of out-of-school mathematics in relation to her students lived experiences. Rather, her example of out-of-school mathematics was demography or geography.

Power

A key component of this dimension is that teachers see the mathematical potential of all of their students, allow them to take ownership of their ideas, and position them as experts in the classroom. As we were discussing the concepts of positioning students as experts, I asked Elizabeth to recall and example she previously shared with me of a student who counted and tapped her chin as she counted to keep track of the number. I suggested to Elizabeth that this could be an example where she could allow this student to share her counting method with the class. Elizabeth responded:

Usually they can't express it because they don't have . . . They don't know what they're doing. They've just seen adults do it and they think that's counting...I assume some type of counting method that is used in that country, but I usually would ask them about it and they could continue doing that if it helped them But I wouldn't encourage the rest of the class to start doing it too. I just said this is the way we're going to do it here and if that's the way you do it, you can do that, that's fine.

I asked Elizabeth if she could think of an example when she would position students as experts and allow them to take ownership of ideas in the classroom. She provided the following example:

We just had a center – a subtraction center – that was manned by Paul, Natalie and Jared. Natalie had her first grade workbooks and she asked me if she could have a center during math time that

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if people were done they would be able to visit the math center and practice subtraction because she knew how to do it and she wanted to teach them how to do it.

Later in this conversation I asked Elizabeth to think of other students that she could position as experts or different types of expertise that existed in her classroom. She shared another example about Paul. These examples demonstrate Elizabeth's belief that only some students can be seen as experts in the classroom. The student referenced in the first example was one of Elizabeth's "lower" students. Paul, Natalie, and Jared on the other hand were her top three students. Throughout the study, I asked Elizabeth to share examples of students taking ownership of ideas in the classroom or demonstrating expertise. For Elizabeth, only her "high" students were referenced. These examples indicate that the trajectory was not enough to help Elizabeth see all students as possessors or knowledge therefore allowing all students to be experts.

Discussion

My goal for this paper was to present an important "unintended consequence" that emerged as a result of a professional development project. While other teachers in the case study exhibited growth/progress/demonstrated change, Elizabeth did not. Using the LT as a tool to focus on the students' individual thinking did not disrupt Elizabeth's beliefs about who could and could not do mathematics. While Elizabeth did demonstrate findings from other studies of LTs (e.g., using LTs to set goals for students), these findings did not cut across all subgroups of students in her class.

Previous work has shown promise that when content-focused PD and issues of culture, privilege and power are fully integrated; teachers can begin to acknowledge the contributions of all students and deficit orientations can be dispelled (Battey & Chan, 2010). Elizabeth's case presents a critical challenge for the field. When teachers have strong deficit perspectives, those beliefs will carry over to their professional development experiences. Therefore, although teachers may engage in new and innovative pedagogies, not all students will benefit from this new professional knowledge. This case highlights the fact that although Elizabeth was able to use elements of the LTBI in her instruction, they were not implemented equitably. Therefore, to avoid unintended consequences of content-only focused PD, teacher educators must consider simultaneously addressing issues of culture, power, and privilege.

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